

$$\int \ln \sqrt{x-1} dx = \overbrace{x \ln \sqrt{x-1}}^{uv} - \int \underbrace{x \frac{1}{2\sqrt{x-1}}}_{vdu} dx$$

$$u = \ln \sqrt{x-1} \Rightarrow du = \frac{1}{2\sqrt{x-1}}$$

$$dv = dx \Rightarrow v = x$$

$$= x \ln \sqrt{x-1} - \int \frac{x dx}{2(x-1)}$$

$$= x \ln \sqrt{x-1} - \int \frac{(x-1) dx}{2(x-1)} - \int \frac{dx}{2(x-1)}$$

$$= x \ln \sqrt{x-1} - \frac{x}{2} - \frac{1}{2} \ln |x-1| + C$$

$$= (x-1) \ln \sqrt{x-1} - \frac{x}{2} + C$$

$$\int \frac{3x-7}{(x-1)(x-2)(x-3)} dx$$

$$\frac{3x-7}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3}$$

$$A = \frac{-4}{(-1)(-2)} = -2, \quad B = \frac{-1}{(1)(-1)} = 1, \quad C = \frac{2}{2 \cdot 1} = 1$$

$$\Rightarrow \int \frac{3x-7}{(x-1)(x-2)(x-3)} dx = -2 \int \frac{dx}{x-1} + \int \frac{dx}{x-2} + \int \frac{dx}{x-3}$$

$$= -2 \ln |x-1| + \ln |x-2| + \ln |x-3| + C$$

$$= \ln \frac{(x-2)(x-3)}{(x-1)^2} + C$$

$$\int \frac{dx}{x \ln x (\ln x - 1)} = \int \frac{du}{u(u+1)}$$

$$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1} = \frac{A(u+1) + Bu}{u(u+1)}$$

$$A+B=0, A=1 \Rightarrow B=-1$$

$$\ln x - 1 = u \Rightarrow$$

$$\frac{dx}{x} = du$$

$$= \int \frac{du}{u} - \int \frac{du}{u+1} = \ln|u| - \ln|u+1| + C$$

$$= \ln \left| \frac{u}{u+1} \right| + C$$

$$= \ln \left| \frac{\ln x - 1}{\ln x} \right| + C$$

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$$\int dx \quad - \int \frac{2dz}{1-z^2} \quad \int \frac{2dz}{1-z^2} \quad \int \frac{2(1-z^2)dz}{(1-z^2)^2} \quad \int \frac{2(1-z^2)dz}{(1-z^2)^2}$$

$$\int \frac{dx}{\sqrt{2x-x^2}} = \int \frac{dx}{\sqrt{1-(x-1)^2}} = \int \frac{du}{\sqrt{1-u^2}} = \sin^{-1} u + C = \sin^{-1}(x-1) + C$$

$$\int \frac{dx}{4x^2+4x+2} = \int \frac{dx}{4(x^2+x)+2}$$

$$= \int \frac{dx}{4\left(\frac{x+1/2}{1}\right)^2+1} = \int \frac{du}{4u^2+1}$$

$$= \frac{1}{4} \int \frac{du}{u^2+1/4} = \frac{1}{4} \tan^{-1} \frac{x+1/2}{1/2} + C$$

$$= \frac{1}{4} \tan^{-1}(2x+1) + C$$

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$$\int \tan x \sin x dx = -\tan x \cos x + \int \cos x \sec^2 x dx$$

$$\begin{cases} \tan x = u \\ \sin x dx = du \\ \sec^2 x dx = du \\ -\cos x = v \end{cases} \quad \begin{aligned} &= -\tan x \cos x + \int \frac{\cos x dx}{\cos^2 x} \\ &= -\tan x \cos x + \int \sec x dx \\ &= -\tan x \cos x + \ln|\sec x + \tan x| + C \end{aligned}$$

$$\int \frac{\cot x dx}{3+2\sin x} = \int \frac{\cos x dx}{3\sin x + 2\sin^2 x} = \int \frac{du}{3u+2u^2} = \frac{1}{3} \int \frac{du}{u} - \frac{2}{3} \int \frac{du}{3+2u}$$

$$\sin x = u \Rightarrow \cos x dx = du$$

$$\frac{1}{3u+2u^2} = \frac{A}{u} + \frac{B}{3+2u} = \frac{3A+2Au+Bu}{u(3+2u)}$$

$$2A+B=0, 3A=1 \Rightarrow A=\frac{1}{3}, B=-\frac{2}{3}$$

$$\begin{aligned} &= \frac{1}{3} \ln|u| - \frac{2}{3} \ln|3+2u| + C \\ &= \frac{1}{3} \ln|\sin x| - \frac{2}{3} \ln|3+2\sin x| + C \\ &= \frac{1}{3} \ln \left| \frac{\sin x}{3+2\sin x} \right| + C \end{aligned}$$

$$\begin{aligned} \int \cos x \ln \sin x dx &= \int \ln u du = u(\ln u - 1) + C \\ &= \sin x (\ln(\sin x) - 1) + C \end{aligned}$$

$$\sin x = u$$

$$\cos x dx = du$$

مر ۱.۲
لاستیک

راه حل اول - ۲۳

$$\int \frac{2 + \ln^2 x}{x(1 - \ln x)} dx \Rightarrow$$

$$\ln x = u \Rightarrow du = \frac{1}{x} dx$$

$$= \int \frac{2 + \ln^2 x + \ln x - \ln x}{x(1 - \ln x)} dx = \int \frac{-\ln x(1 - \ln x) dx}{x(1 - \ln x)} \quad \underbrace{u'}$$

$$+ \int \frac{(2 + \ln x) dx}{x(1 - \ln x)} = -\frac{1}{2} \ln^2 x + \int \frac{2 dx}{x(1 - \ln x)} + \int \frac{\ln x dx}{x(1 - \ln x)} \quad \underbrace{u}$$

$$= -\frac{1}{2} \ln^2 |x| + 2 \int \frac{du}{(1-u)} - \ln |x(1 - \ln x)| + C$$

$$= -\frac{1}{2} \ln^2 |x| - 2 \ln |1-u| - \ln |x(1 - \ln x)|$$

$$= -\frac{1}{2} \ln^2 x - 2 \ln |1 - \ln x| - \ln |x(1 - \ln x)| + C$$

$$\int \frac{2 + \ln^2 x}{1 - \ln x} \cdot \frac{1}{x} dx$$

راه حل دوم = ۲۴

$$\ln x = u \Rightarrow \frac{1}{x} dx = du$$

اگر توان صورت مساوی با برابر از توان خارج بود قسمتی نسیم

$$\frac{u^2 + 2}{u^2 - u} \left| \frac{-u+1}{-u-1} \right|$$

$$\frac{u+2}{u-1}$$

$$3$$

$$\int \frac{2 + \ln^2 x}{x(1 - \ln x)} dx =$$

$$\int \frac{2 + u^2}{1 - u} du = \int \left[(-u-1) + \left(\frac{3}{-u+1} \right) \right] du$$

$$] du = -\frac{u^2}{2} - u - 3 \ln |-u+1| + C$$

$$= -\frac{\ln^2 x}{2} - \ln x - 3 \ln |-\ln x + 1| + C$$

$$\int \frac{e^x dx}{\sin^2 e^x (csc^2 e^x - 3cot e^x + 1)} = \int \frac{dt}{\sin^2 t (csc^2 t - 3cot t + 1)}$$

$$e^x = t$$

$$e^x dx = dt$$

$$= \int \frac{csc^2 t dt}{(csc^2 t - 3cot t + 1)} = \text{بقدر عوامل فخرج جيب التمام}$$

$$= \int \frac{csc^2 t dt}{(1 + cot^2 t - 3cot t + 1)} = \int \frac{csc^2 t dt}{cot^2 t - 3cot t + 2}$$

$$cot t = u \rightarrow -du = csc^2 t dt$$

$$= - \int \frac{du}{u^2 - 3u + 2} = - \int \frac{du}{(u-1)(u-2)}$$

$$\frac{1}{(u-1)(u-2)} = \frac{A}{u-1} + \frac{B}{u-2} = \frac{Au - 2A + Bu - B}{(u-1)(u-2)}$$

$$(A+B)u - 2A - B = 1$$

$$A+B=0 \rightarrow A=-B$$

$$-2A - B = 1 \rightarrow -2A + A = 1 \rightarrow A = -1 \quad B = 1$$

$$= \int \frac{du}{u-1} - \int \frac{du}{u-2} = \ln|u-1| - \ln|u-2| + C$$

ادامه تغییر متغیرها:

از این تغییر متغیرها: در انتگرالها رشت مثل کرباع مثلثاتی استفاده از تغییر متغیر زیر ممکن است به حل انتگرال کمک کند.

$$z = \tan \frac{x}{2} \quad \cos x = \cos^2 \frac{x}{2} - \sin^2 \frac{x}{2} \quad \cos x = \frac{1-z^2}{1+z^2}$$

$$1 + \tan^2 \frac{x}{2} = \sec^2 \frac{x}{2} \quad \frac{1}{\cos^2 \frac{x}{2}} = z^2 + 1$$

$$dz = \frac{1}{2} \sec^2 \frac{x}{2} dx \quad \frac{1}{\cos^2 \frac{x}{2}} = z^2 + 1 \quad \cos^2 \frac{x}{2} = \frac{1}{z^2 + 1} \quad \sin x = \frac{2z}{1+z^2}$$

$$dz = \frac{1}{2} (z^2 + 1) dx \quad \sin^2 \frac{x}{2} = 1 - \frac{1}{z^2 + 1} = \frac{z^2}{z^2 + 1} \quad \cos x = \frac{1}{z^2 + 1} - \frac{z^2}{z^2 + 1} = \frac{1-z^2}{1+z^2}$$

$$dx = \frac{2 dz}{1+z^2} = \frac{2 dz}{z^2 + 1} \quad \tan x = \frac{2z}{1-z^2}$$

اینز برابر حالتی است که $\cos x$ بار در ج ۱ موجود است.

$$\int \frac{dx}{5 + \cos x} = \int \frac{\frac{2 dz}{1+z^2}}{5 + \frac{1-z^2}{1+z^2}} \quad \text{مثال ۳}$$

$$z = \tan \frac{x}{2}$$

$$\cos x = \frac{1-z^2}{1+z^2}$$

$$= \int \frac{2 dz}{5 + 5z^2 + 1 - z^2} = \int \frac{2 dz}{4z^2 + 6} - \int \frac{dz}{2z^2 + 3}$$

$$= \frac{1}{2} \int \frac{dz}{z^2 + 3/2} = \frac{1}{2} \frac{1}{\sqrt{3/2}} \tan^{-1} \frac{z}{\sqrt{3/2}} + C$$

$$\frac{1}{a} \tan^{-1} \frac{1}{a}$$

$$= \frac{1}{\sqrt{3}} \tan^{-1} \frac{\tan \frac{x}{2}}{\sqrt{3/2}} + C$$

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$$\int \frac{\ln(x^2+1) dx}{u} \frac{dv}{dv} = x \ln(x^2+1) - \int \frac{2x^2 dx}{x^2+1} = x \ln(x^2+1) - 2x + 2 \tan^{-1} x + C$$

$$du = \frac{2x}{x^2+1} dx$$

$$v = x$$

$$\int \frac{dx}{2+2\sin x + \cos x} = \int \frac{\frac{2dz}{1+z^2}}{2+2\frac{2z}{1+z^2} + \frac{1-z^2}{1+z^2}}$$

$$\frac{1}{3+4z+z^2} = \frac{A}{z+1} + \frac{B}{z+3} = \frac{A(z+3) + B(z+1)}{(z+1)(z+3)}$$

$$z = \tan x/2$$

$$= \int \frac{2dz}{2+2z^2+4z+1-z^2}$$

$$\begin{aligned} A+B &= 0 \\ 3A+B &= 1 \end{aligned} \Rightarrow A = 1/2, B = -1/2$$

$$= \int \frac{2dz}{3+4z+z^2}$$

$$= \int \frac{dz}{z+1} - \int \frac{dz}{z+3} = \ln \left| \frac{z+1}{z+3} \right| + C$$

$$= \ln \left| \frac{1+\tan x/2}{3+\tan x/2} \right| + C$$

$$\begin{aligned} \int \frac{dt}{t-\sqrt{1-t^2}} &= \int \frac{(t+\sqrt{1-t^2})dt}{t^2-1+t^2} = \int \frac{tdt}{2t^2-1} + \int \frac{\sqrt{1-t^2}dt}{2t^2-1} \\ &= \frac{1}{4} \ln |2t^2-1| \end{aligned}$$

دنباله تغییر متغیرها: (378) $\sqrt[n]{(ax+b)^m}$ (در صورتی که $a \neq 0$)

اگر فرض شود $u^n = (ax+b)$

$$u^n = (ax+b)$$

$$(ax+b)^{m/n} = (u^n)^{m/n} = u^m$$

$$\Rightarrow \sqrt[n]{(ax+b)^m} = (ax+b)^{m/n} = u^m$$

مثال:

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$$\int \frac{\sqrt{x} dx}{1 + \sqrt[4]{x}}$$

فرض شود

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مثال: $\int \frac{\sqrt{x} dx}{1 + \sqrt[4]{x}}$ $u = \sqrt[4]{x} \rightarrow x = u^4 \rightarrow dx = 4u^3 du$

$$u = \sqrt[4]{x} \rightarrow x = u^4 \rightarrow dx = 4u^3 du$$

$$= \int \frac{u^2 \cdot 4u^3 du}{1+u} = 4 \int \frac{u^5 du}{1+u}$$

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بقیم تغییر متغیرها

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$$u = t^{n/m}, \quad u^m = t^n, \quad \sqrt[n]{u^m} \quad x \text{ تعویض}$$

$$\int \frac{1 + \sqrt[3]{x}}{\sqrt[6]{x}} dx = \int \frac{1 + u^2}{u} (6u^5) du = 6 \int (u^4 + u^6) du$$

$$= \frac{6u^5}{5} + \frac{6u^7}{7} + C \quad x = u^6 \quad dx = 6u^5 du$$

فرمول های کسری توان

#34
p.426

$$\int \frac{x^n e^x dx}{u} = x^n e^x - n \int x^{n-1} e^x dx$$

$$\begin{cases} u = x^n \Rightarrow du = nx^{n-1} dx \\ e^x dx = dv \Rightarrow e^x = v \end{cases}$$

$$\int x^n e^x dx = x^n e^x - \int nx^{n-1} e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

$$\begin{aligned} \int_0^1 x^3 e^x dx &= x^3 e^x \Big|_0^1 - 3 \int_0^1 x^2 e^x dx \\ &= x^3 e^x \Big|_0^1 - 3 \left(x^2 e^x \Big|_0^1 - 2 \int_0^1 x e^x dx \right) \\ &= x^3 e^x \Big|_0^1 - 3 x^2 e^x \Big|_0^1 + 6 \left(x e^x \Big|_0^1 - \int_0^1 e^x dx \right) \\ &= e - 3e + 6e - 6e \Big|_0^1 \\ &= 4e - 6e + 6 = 6 - 2e \end{aligned}$$

#33
p.445

$$\int x^m (\ln x)^n dx = \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx$$

$$\begin{cases} (\ln x)^n = u \Rightarrow du = n/x (\ln x)^{n-1} dx \\ x^m dx = dv \Rightarrow v = \frac{x^{m+1}}{m+1} \end{cases}$$

$$\begin{aligned} &= \frac{x^{m+1} (\ln x)^n}{m+1} - \int \frac{x^{m+1}}{m+1} \cdot \frac{n (\ln x)^{n-1}}{x} dx \\ &= \frac{x^{m+1} (\ln x)^n}{m+1} - \frac{n}{m+1} \int x^m (\ln x)^{n-1} dx \end{aligned}$$

$$\int_1^3 x^3 (\ln x)^2 dx = \frac{x^4 (\ln x)^2}{4} \Big|_1^3 - \frac{2}{4} \int_1^3 x^3 (\ln x) dx = \frac{3^4 (\ln 3)^2}{4} - \frac{1}{2} I$$

$$= 3^4 (\ln 3)^2 / 4 - 1/2 \left(\frac{3^4 \ln 3}{4} - \frac{3^4}{4^2} - \frac{1}{16} \right)$$

$$\begin{aligned} \int_1^3 x^3 \ln x dx &= \frac{x^4 \ln x}{4} \Big|_1^3 - \frac{1}{4} \int_1^3 x^3 dx \\ &= \frac{3^4 \ln 3}{4} - \frac{1}{4} \left(\frac{x^4}{4} \Big|_1^3 \right) \\ &= \frac{3^4 \ln 3}{4} - \frac{3^4}{4^2} + \frac{1}{16} \end{aligned}$$