

$$4. \int 16x \sin^3(2x^2+1) \cos(2x^2+1) dx = \int 4 \sin^3 u \cos u du$$

$$= \sin^4 u + C = \sin^4(2x^2+1) + C$$

sol: $u = 2x^2 + 1, du = 4x dx$

sol: $u = \sin(2x^2+1), du = \cos(2x^2+1) \cdot 4x dx$ Y Y

$$4 du = 16x \cos(2x^2+1) dx$$

$$= \int 4 u^3 du = u^4 + C = \sin^4(2x^2+1) + C$$

$$5. \int \sin^n x \cos x dx, n \neq -1$$

$u = \sin x, du = \cos x dx$

$$= \int u^n du = \frac{u^{n+1}}{n+1} + C = \frac{\sin^{n+1} x}{n+1} + C$$

$$6. \int \cos^n x \sin x dx, n \neq -1$$

$u = \cos x, du = -\sin x dx$

$$= \int u^n (-du) = -\frac{u^{n+1}}{n+1} + C = -\frac{\cos^{n+1} x}{n+1} + C$$

$$n = -1 \Rightarrow \int (\sin x)^{-1} \cos x dx = \int \cot x dx = \ln |\sin x| + C$$

$$\int (\cos x)^{-1} \sin x dx = \int \tan x dx = -\ln |\cos x| + C$$

انٲرال تآبآى سٲٲى دكٲر ✓

$$\int \sec^2 u du = \tan u + C$$

$$\int \sec u \tan u du = \sec u + C$$

$$\int \csc^2 u du = -\cot u + C$$

$$\int \csc u \cot u du = -\csc u + C$$

$$7. a) \int \frac{1}{\cos^2 2x} dx = \int \sec^2 2x dx = \frac{1}{2} \int \sec^2 2x d(2x) = \frac{1}{2} \tan 2x + C$$

$$b) \int \frac{\sin x}{\cos^2 x} dx = \int \frac{1}{\cos x} \cdot \frac{\sin x}{\cos x} dx = \int \sec x \tan x dx = \sec x + C$$

$$c) \int \sec x \tan x dx = \int \sec x (\sec x \tan x) dx = \int \sec x d(\sec x)$$

$$= \frac{1}{2} \sec^2 x + C$$

$$8. \int \tan^3 x \sec^2 x dx = \int u^3 du = \frac{1}{4} u^4 + C = \frac{1}{4} \tan^4 x + C$$

$$u = \tan x, du = \sec^2 x dx$$

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ان شاء الله تعالى (Inshallah)

$$9. \int \tan^2 x dx = \int (\sec^2 x - 1) dx = \int \sec^2 x dx - \int dx = \tan x - x + C$$

$$10. \int \cos^2 x dx = \int \frac{1 + \cos 2x}{2} dx = \int \frac{1}{2} dx + \int \frac{\cos 2x}{2} dx$$

$$= \frac{x}{2} + \frac{1}{2} \frac{\sin 2x}{2} + C = \frac{x}{2} + \frac{\sin 2x}{4} + C$$

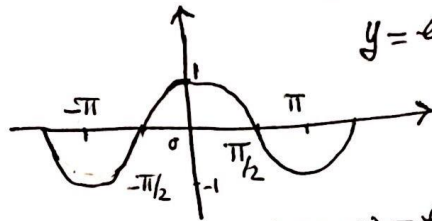
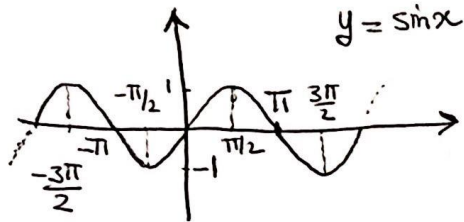
$$11. \int \sin^2 x dx = \int \frac{1 - \cos 2x}{2} dx = \int \frac{1}{2} dx - \int \frac{\cos 2x}{2} dx$$

$$= \frac{x}{2} - \frac{\sin 2x}{4} + C$$

$$12. \int \sin^3 x dx = \int (1 - \cos^2 x) \sin x dx = \int \sin x dx - \int \cos^2 x \sin x dx$$

$$= -\cos x + \frac{1}{3} \cos^3 x + C$$

مغودار تابع سینوس و کینوس



$$\sin(2\pi + x) = \sin x$$

$$\cos(2\pi + x) = \cos x$$

تابع یک به یک نیست

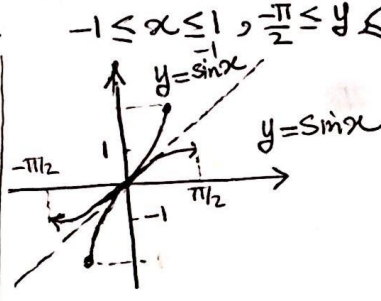
این دو تابع را در نظر بگیرید.

$$\left\{ \begin{array}{l} g(x_1) = g(x_2) \Rightarrow x_1 = x_2 \\ x_1 \neq x_2 \Rightarrow g(x_1) \neq g(x_2) \end{array} \right.$$

اما در بازه $[-\pi/2, \pi/2]$ یکتا، متعین و اکیدا متعین است پس معکوس دارد.

$$y = \sin x, \quad -\pi/2 \leq x \leq \pi/2, \quad -1 \leq y \leq 1$$

$$\begin{aligned} -1 &\leq y = \sin x \leq 1 \\ \Rightarrow 0 &\leq x^2 \leq 2 \\ \Rightarrow x^2 &\leq 2 \\ \Rightarrow |x| &\leq \sqrt{2} \\ \Rightarrow -\sqrt{2} &\leq x \leq \sqrt{2} \end{aligned}$$



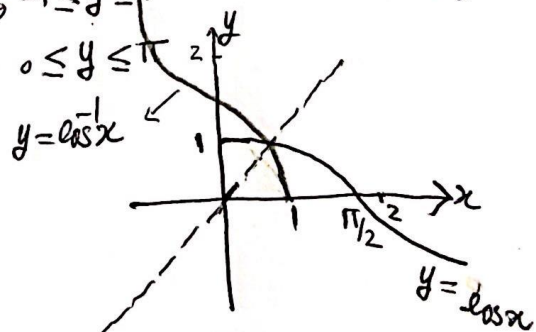
$$\begin{aligned} y = \sin x &\Leftrightarrow \sin y = x \\ \Rightarrow (\cos y) y' &= 1 \\ \Rightarrow y' &= \frac{1}{\cos y} \end{aligned}$$

$$\sin^2 y + \cos^2 y = 1 \quad \text{اما} \quad \cos y = \sqrt{1 - \sin^2 y} \quad \text{پس} \quad \cos y = \sqrt{1 - x^2}$$

$$\sin^{-1} u = \frac{u'}{\sqrt{1-u^2}} \quad \text{در حالت کلی} \quad (\sin^{-1} x)' = \frac{1}{\sqrt{1-x^2}}$$

(2) $f(x) = \cos x$ در بازه $[0, \pi]$ اکیدا متعین و متعین است پس یک به یک است. معکوس دارد.

$$\begin{aligned} y = \cos x, \quad 0 \leq x \leq \pi, \quad -1 \leq y \leq 1 \\ y = \cos^{-1} x, \quad -1 \leq x \leq 1, \quad 0 \leq y \leq \pi \end{aligned}$$



مشتق : $y = \cos^{-1} x \Rightarrow x = \cos y$
 $\Rightarrow 1 = -(\sin y) y'$
 $\Rightarrow y' = \frac{1}{-\sin y}$

اما $\sin^2 y = 1 - \cos^2 y$
 و چون $0 \leq y \leq \pi$ و $\sin y > 0$ لذا $\sin y = \sqrt{1 - \cos^2 y}$
 پس $\sin y = \sqrt{1 - x^2}$

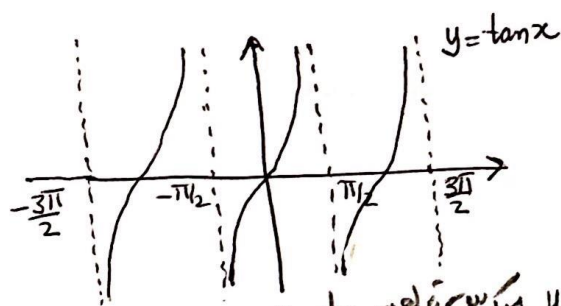
$$y' = \frac{1}{-\sin y} = \frac{1}{-\sqrt{1-x^2}}$$

پس $(\cos^{-1} u)' = -\frac{u'}{\sqrt{1-u^2}}$ و در صورت کلی $(\cos^{-1})' = \frac{-1}{\sqrt{1-x^2}}$
 و تابعی از x است.

نتیجه: (1) $\int \frac{1}{\sqrt{1-x^2}} dx = \sin^{-1} x + C$ و $\int \frac{-1}{\sqrt{1-x^2}} = \cos^{-1} x + C$

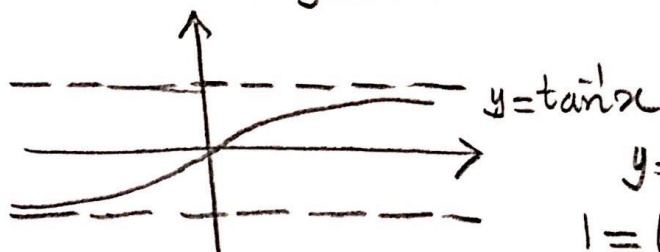
(2) $(\sin^{-1} x)' + (\cos^{-1} x)' = 0 \Rightarrow \sin^{-1} x + \cos^{-1} x = C$
 که C عدد ثابت است.

$x = 0, \sin^{-1} 0 = 0, \cos^{-1} 0 = \pi/2 \Rightarrow \sin^{-1} x + \cos^{-1} x = \pi/2$



(3) $f(x) = \tan x$
 اگر دامنه را به $(-\pi/2, \pi/2)$ محدود کنیم
 در این صورت تابع اکیدا صعودی و لذا
 یک به یک است.
 $y = \tan x, -\pi/2 < x < \pi/2, -\infty < y < \infty$

معکوس $y = \tan^{-1} x, -\infty < x < \infty, -\pi/2 < y < \pi/2$



مشتق $y = \tan^{-1} x$

$y = \tan^{-1} x \Rightarrow x = \tan y$

مشتق می گیریم: $1 = (1 + \tan^2 y) y'$

پس $y' = \frac{1}{1 + \tan^2 y} = \frac{1}{1 + x^2}$

و در صورت کلی: $(\tan^{-1} u)' = \frac{u'}{1+u^2}$

$$F(x) = \ln(\tan^{-1} 3x), \quad F'(x) = ?$$

حل

$$F'(x) = \frac{(\tan^{-1} 3x)'}{\tan^{-1} 3x} = \frac{\frac{3}{1+9x^2}}{\tan^{-1} 3x} = \frac{3}{(1+9x^2)\tan^{-1} 3x}$$

$$\sin y + x y' \cos y + 3x^2 = \tan y \quad \text{حل}$$

$$\sin y + x y' \cos y + 3x^2 = \frac{y'}{\tan y} \Rightarrow$$

$$\sin y \tan y + x y' \cos y \tan y + 3x^2 \tan y = y'$$

$$y'(1 - x \cos y) = \sin y \tan y + 3x^2 \tan y$$

$$\Rightarrow y' = \frac{\sin y \tan y + 3x^2 \tan y}{1 - x \cos y}$$

$$\int \frac{1}{1+x^2} dx = \tan^{-1} x + C, \quad \int \frac{du}{1+u^2} = \tan^{-1} u + C$$

حل

$$\int \frac{x dx}{7+e^{2x}} = \int \frac{du}{7+u^2} = \frac{1}{7} \int \frac{du}{1+(\frac{u}{\sqrt{7}})^2} = \frac{1}{\sqrt{7}} \int \frac{\frac{du}{\sqrt{7}}}{1+(\frac{u}{\sqrt{7}})^2}$$

$$e^x = u \Rightarrow e dx = du$$

$$= \frac{1}{\sqrt{7}} \tan^{-1} \frac{u}{\sqrt{7}} + C$$

$$= \frac{1}{\sqrt{7}} \tan^{-1} \frac{e^x}{\sqrt{7}} + C$$

$$\int \frac{dx}{(1+x)\sqrt{x}} = \int \frac{2udu}{(1+u^2)u} = 2 \int \frac{du}{1+u^2} = 2 \tan^{-1} u + C$$

حل

$$= 2 \tan^{-1} \sqrt{x} + C$$

$$\sqrt{x} = u \Rightarrow x = u^2$$

$$dx = 2u du$$

$$\int \frac{dx}{\sqrt{15+2x-x^2}} = \int \frac{dx}{\sqrt{-(x^2-2x+1)+16}}$$

حل

$$= \int \frac{dx}{\sqrt{4-(x-1)^2}} = \int \frac{dx}{2\sqrt{1-(\frac{x-1}{2})^2}}$$

$$= \int \frac{\frac{dx}{2}}{\sqrt{1-(\frac{x-1}{2})^2}} = \sin^{-1} \left(\frac{x-1}{2} \right) + C$$

