

$$1. \frac{dy}{dx} = \frac{x^3 - 2y}{x}$$

$$y' = x^2 - \frac{2}{x}y \Rightarrow y' + \frac{2}{x}y = x^2 \quad \mu(x) = \exp\left(\int \frac{2}{t} dt\right) = \exp \ln x^2$$

$$\mu(x) = x^2 \Rightarrow x^2 y' + 2xy = x^4 \Rightarrow (x^2 y)' = x^4 \Rightarrow x^2 y = \frac{x^5}{5} + C$$

$$\therefore y = \frac{x^3}{5} + \frac{C}{x^2}$$

$$2. (x+y)dx - (x-y)dy = 0$$

$$\frac{dy}{dx} = \frac{x+y}{x-y} \quad \frac{y}{x} = u \Rightarrow y = xu \Rightarrow y' = xu' + u$$

$$\therefore xu' + u = \frac{x+xu}{x-xu} \Rightarrow xu' + u = \frac{1+u}{1-u} \Rightarrow xu' = \frac{1+u-u(1-u)}{1-u}$$

$$\Rightarrow x \frac{du}{dx} = \frac{1+u^2}{1-u} \Rightarrow \frac{1-u}{1+u^2} du = \frac{dx}{x} \Rightarrow \int \frac{1-u}{1+u^2} du = \int \frac{1}{x} dx$$

$$\ln|x| + C \Rightarrow \int \frac{1-u}{1+u^2} du = \frac{1}{2} \ln|1+u^2| - \frac{1}{2} \ln|1+u^2| = C$$

$$\Rightarrow \frac{1}{2} \ln|1+u^2| - \frac{1}{2} \ln|x(1+u^2)| = C \Rightarrow \frac{1}{2} \ln \frac{1+u^2}{x(1+u^2)} = C$$

$$\Rightarrow \frac{1}{2} \ln \frac{1}{x} = C \Rightarrow \frac{1}{2} \ln \frac{1}{x} = C$$

$$x dx + y dx + y dy - x dy = 0 \Rightarrow \frac{1}{2} d(x^2 + y^2) + y dx - x dy = 0$$

$$\Rightarrow \frac{1}{2} \frac{d(x^2 + y^2)}{x^2 + y^2} + \frac{y dx - x dy}{x^2 + y^2} = 0 \Rightarrow \ln \sqrt{x^2 + y^2} + \frac{1}{2} \frac{y dx - x dy}{x^2 + y^2} = C$$

$$3. \frac{dy}{dx} = \frac{2x+y}{3+3y^2-x}, y(0) = 0$$

$$(2x+y)dx + (x-3y^2-3)dy = 0$$

$$M(x,y) = 2x+y, N(x,y) = x-3y^2-3$$

$$M_y(x,y) = 1, N_x(x,y) = 1 \quad M_y(x,y) = N_x(x,y) \Rightarrow \text{D.E. is exact.}$$

$$\psi_x(x, y) = 2x + y, \quad \psi_y(x, y) = x - 3y^2 - 3$$

$$\psi(x, y) = x^2 + xy + h(y) \quad \psi_y(x, y) = x + h'(y) = x - 3y^2 - 3$$

$$h'(y) = -3y^2 - 3 \Rightarrow h(y) = -y^3 - 3y$$

$$\psi(x, y) = x^2 + xy - y^3 - 3y \Rightarrow x^2 + xy - y^3 - 3y = C$$

$$\& C=0 \Rightarrow x^2 + xy - y^3 - 3y = 0$$

$$4) (x + e^y) dy - dx = 0$$

$$M(x, y) = 1, \quad N(x, y) = -(x + e^y)$$

$$M_y = 0, \quad N_x = -1 \quad \frac{N_x - M_y}{M} = \frac{-1}{1} = -1 = Q(y)$$

$$\mu(y) = \exp \int Q(y) dy = \exp \int (-1) dy = \exp(-y)$$

$$\Rightarrow -e^{-y} dx + (xe^{-y} + 1) dy = 0 \Rightarrow d(-xe^{-y} + y) = 0 \Rightarrow$$

$$-xe^{-y} + y = C$$

(با روش Exact حل شد)

$$5. \frac{dy}{dx} = \frac{2xy + y^2 + 1}{x^2 + 2xy}$$

$$(2xy + y^2 + 1) dx + (x^2 + 2xy) dy = 0$$

$$d(x^2y + xy^2 + x) = 0 \Rightarrow x^2y + xy^2 + x = C$$

Exact (با روش)

$$6. x \frac{dy}{dx} + xy = 1 - y, \quad y(1) = 0$$

$$x \frac{dy}{dx} + (x+1)y = 1 \Rightarrow y' + (1 + \frac{1}{x})y = \frac{1}{x}$$

$$\mu(x) = \exp \int (1 + \frac{1}{x}) dt = \exp(x + \ln x) = xe^x$$

$$xe^x y' + (x+1)e^x y = e^x \Rightarrow (xe^x y)' = e^x \Rightarrow xe^x y = e^x + C$$

$$\Rightarrow y = \frac{1}{x} + C \frac{e^{-x}}{x}, \quad 0 = e^1 + C \Rightarrow C = -e$$

$$y = \frac{1}{x} (1 - e^{1-x})$$

$$7. \frac{dy}{dx} = \frac{x}{x^2y + y^3}$$

$$\text{Hint: let } u = x^2$$

$$x^2 + y^2 = u$$

راه دیگر: جدا کردن متغیرها

مقدار معادله دیفرانسیل

$$u = x^2 \Rightarrow 2x dx = du \quad \frac{dy}{du} = \frac{1}{2(uy + y^3)}$$

$$\frac{du}{dy} = 2(uy + y^3) \Rightarrow u' - 2uy = 2y^3 \quad \mu(y) = \exp \int \frac{dy}{y^2} = e^{-\frac{1}{y}}$$

$$\begin{aligned} e^{-\frac{1}{y}} u' - 2y e^{-\frac{1}{y}} u &= 2e^{-\frac{1}{y}} y^3 \Rightarrow (e^{-\frac{1}{y}} u)' = 2y^3 e^{-\frac{1}{y}} \\ \int 2y^3 e^{-\frac{1}{y}} dy &= dv \quad \int 2y^3 e^{-\frac{1}{y}} dy = -y^2 e^{-\frac{1}{y}} - e^{-\frac{1}{y}} + C \\ u_1 &= y^2 \end{aligned}$$

$$(x^2 + y^2 + 1) e^{-\frac{1}{y}} = C$$

$$8. x \frac{dy}{dx} + 2y = \frac{\sin x}{x} \quad \mu(x) = \exp \int \frac{2}{x} = x^2 \quad (y(0) = 1)$$

$$\frac{dy}{dx} + \frac{2}{x} y = \frac{\sin x}{x^2} \Rightarrow x^2 \frac{dy}{dx} + 2xy = \sin x$$

$$\Rightarrow (x^2 y)' = \sin x \Rightarrow x^2 y = -\cos x + C \quad 4 + 4 = -\cos 2 + C$$

$$\Rightarrow y = x^{-2} (4 + \cos 2 - \cos x)$$

$$9. \frac{dy}{dx} = -\frac{2xy + 1}{x^2 + 2y}$$

$$(2xy + 1)dx + (x^2 + 2y)dy = 0 \Rightarrow d(x^2 y + x + y^2) = 0$$

$$\Rightarrow x^2 y + x + y^2 = C$$

$$M_y = 2x, N_x = 2x \quad M_y = N_x \therefore \text{eq. is exact, prob}$$

$$\psi_x = 2xy + 1 \quad \& \quad \psi_y = x^2 + 2y \quad (*)$$

$$\psi = x^2 y + x + h(y) \Rightarrow \psi_y = x^2 + h'(y) = x^2 + 2y \Rightarrow h'(y) = 2y$$

$$h(y) = y^2 \quad \psi(x, y) = x^2 y + x + y^2 \therefore x^2 y + x + y^2 = C$$

$$10. (3y^2 + 2xy)dx - (2xy + x^2)dy = 0$$

$$\frac{dy}{dx} = \frac{3y^2 + 2xy}{2xy + x^2} = \frac{3(\frac{y}{x})^2 + 2(\frac{y}{x})}{2(\frac{y}{x}) + 1}$$

$$\frac{y}{x} = u \Rightarrow y = xu \quad y' = x \frac{du}{dx} + u$$

$$\therefore u + x \frac{du}{dx} = \frac{3u^2 + 2u}{2u+1} \Rightarrow x \frac{du}{dx} = \frac{3u^2 + 2u - u(2u+1)}{2u+1}$$

$$x \frac{du}{dx} = \frac{u^2 + u}{2u+1} \Rightarrow \frac{dx}{x} = \frac{2u+1}{u^2+u} du \Rightarrow \ln|ex| = \ln|u^2+u|$$

$$\Rightarrow cx = u^2 + u \Rightarrow cx = \left(\frac{y}{x}\right)^2 + \frac{y}{x} \Rightarrow cx^3 = \frac{y^2}{x} + yx$$

$$\Rightarrow \frac{y^2}{x^3} + \frac{y}{x^2} = c$$

$$11. (x^2 + y)dx + (x + e^y)dy = 0$$

$$M_y = 1, N_x = 1 \quad \text{1st}$$

$$\psi_x = x^2 + y, \quad \psi_y = x + e^y$$

$$\psi = \frac{x^3}{3} + xy + h(y) \Rightarrow \psi_y = x + h'(y) = x + e^y \Rightarrow h(y) = e^y$$

$$h(y) = e^y \Rightarrow \frac{x^3}{3} + xy + e^y = c$$

$$12. \frac{dy}{dx} + y = \frac{1}{1+e^x}$$

$$\mu(x) = \exp \int^x dt = \exp(x) = e^x \quad \text{1st}$$

$$\Rightarrow e^x y' + e^x y = \frac{e^x}{1+e^x} \Rightarrow (e^x y)' = \frac{e^x}{1+e^x} \Rightarrow e^x y = \ln(1+e^x)$$

$$\Rightarrow y = e^{-x} \ln(1+e^x) + ce^{-x}$$

$$13. xdy - ydx = (xy)^{1/2} dx$$

$$((xy)^{1/2} + y)dx - xdy = 0 \Rightarrow \frac{dy}{dx} = \frac{(xy)^{1/2} + y}{x} \quad \text{1st}$$

$$\frac{dy}{dx} = \left(\frac{y}{x}\right)^{1/2} + \frac{y}{x} \quad \frac{y}{x} = u \Rightarrow y = xu \quad \frac{dy}{dx} = x \frac{du}{dx} + u$$

$$x \frac{du}{dx} + u = u^{1/2} + u \Rightarrow x \frac{du}{dx} = u^{1/2}$$

$$\frac{du}{u^{1/2}} = \frac{dx}{x} \Rightarrow \frac{1}{-1/2+1} u^{1/2} = \ln|x| + C \Rightarrow 2u^{1/2} = \ln|x| + C$$

$$\Rightarrow 2\left(\frac{y}{x}\right)^{1/2} = \ln|x| + C$$

$$14. (x+y)dx + (x+2y)dy = 0, \quad y(2) = 3$$

حل: معادله فرق بین معادله حل می باشد
از روش دیگر حل می کنیم

$$\frac{1}{xM+yN} = \frac{1}{x^2+xy+xy+y^2} = \frac{1}{x^2+2xy+y^2} = \frac{1}{(x+y)^2}$$

$$\Rightarrow \frac{x+y}{(x+y)^2} dx + \frac{x+2y}{(x+y)^2} dy = 0 \Rightarrow \frac{dx}{x+y} + \left(\frac{1}{x+y} + \frac{y}{(x+y)^2} \right) dy = 0$$

$$M_y = -\frac{1}{(x+y)^2}, \quad N_x = -\frac{1}{(x+y)^2} - \frac{2(x+y)}{(x+y)^4} =$$

$$15. (e^x+1) \frac{dy}{dx} = y - ye^x$$

$$(e^x+1) \frac{dy}{dx} = y(1-e^x) \quad \frac{dy}{y} = \frac{1-e^x}{1+e^x} dx \quad 1+e^x = u$$

$$\ln|y| = \int \frac{1-(u-1)}{u(u-1)} du = \int \frac{2-u}{u(u-1)} du$$

$$\frac{2-u}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} = \frac{A(u-1)+Bu}{u(u-1)}$$

$$A+B=-1, -A=2 \Rightarrow A=-2, B=1$$

$$\ln|y| = -2\ln|u| + \ln|u-1| + \ln|c| \Rightarrow y = \frac{c(u-1)}{u^2}$$

$$y = \frac{ce^x}{(1+e^x)^2}$$

$$16. \frac{dy}{dx} = \frac{x^2 + y^2}{x^2} = 1 + \left(\frac{y}{x}\right)^2 \quad \frac{y}{x} = u \quad x \frac{du}{dx} + u = 1 + u^2$$

$$\Rightarrow \frac{dx}{x} = \frac{du}{1+u^2-u} = \frac{du}{(u-\frac{1}{2})^2 + \frac{3}{4}} \Rightarrow \ln|x| = \frac{2}{\sqrt{3}} \tan^{-1} \frac{2(u-\frac{1}{2})}{\sqrt{3}} + C$$

$$\therefore \ln|x| = \frac{2}{\sqrt{3}} \tan^{-1} \frac{2}{\sqrt{3}} \left(\frac{y}{x} - \frac{1}{2}\right) + C$$

$$17. \frac{dy}{dx} = e^{2x} + 3y$$

$$\mu(x) = \exp \int -3 dx = e^{-3x} \quad e^{-3x} y - 3e^{-3x} y = e^{-x}$$

$$\Rightarrow (e^{-3x} y)' = e^{-x} \Rightarrow e^{-3x} y = -e^{-x} + C \Rightarrow y = -e^{-x} + Ce^{3x}$$

$$18. (2y + 3x)dx = -x dy$$

$$\frac{dy}{dx} = -\frac{2y+3x}{x} \Rightarrow \frac{dy}{dx} = -2\left(\frac{y}{x}\right) - 3 \quad \frac{y}{x} = v$$

$$x \frac{dv}{dx} + v = -2v - 3 \Rightarrow x \frac{dv}{dx} = -3(v+1)$$

$$\frac{dv}{3(v+1)} = -\frac{dx}{x} \Rightarrow \frac{1}{3} \ln|1+v| = -\ln|x| \Rightarrow \sqrt[3]{1+v} = \frac{1}{cx}$$

$$\Rightarrow 1 + \frac{y}{x} = \frac{1}{(cx)^3} \Rightarrow x+y = cx^{-2}$$

$$19. xdy - ydx = 2x^2 y^2 dy \quad y(1) = -2$$

$$ydx + (2x^2 y^2 - x)dy = 0$$

$$M_y = 1, N_x = 4xy^2 - 1 \quad \frac{M_y - N_x}{N} = \frac{1 - 4xy^2 + 1}{2x^2 y^2 - x}$$

$$= \frac{2 - 4xy^2}{x(2xy^2 - 1)} = -\frac{2}{x} \quad \frac{\mu'(x)}{\mu(x)} = -\frac{2}{x} \Rightarrow \ln \mu(x) = -2 \ln x$$

$$= \ln x^{-2} \Rightarrow \mu(x) = x^{-2} \therefore \frac{y}{x^2} dx + (2y^2 - \frac{1}{x}) dy = 0$$

$$\psi_x = \frac{y}{x^2}, \quad \psi_y = -\frac{1}{x} + 2y$$

$$\psi = -\frac{y}{x} + h(y) \Rightarrow \psi_y = -\frac{1}{x} + h'(y) = 2y - \frac{1}{x} \Rightarrow h'(y) = 2y$$

$$h(y) = \frac{2}{3}y^3 \quad \therefore -\frac{y}{x} + \frac{2}{3}y^3 = C \quad \text{or} \quad 2 + \frac{2}{3}(-8) = C$$

$$\Rightarrow C = -\frac{10}{3} \Rightarrow -3y + 2y^3x = -10x$$

$$20. \quad y' = e^{x+y}$$

$$y' = e^{x+y} = e^x \cdot e^y \Rightarrow \frac{dy}{dx} = e^x \cdot e^y \Rightarrow e^x dx = \frac{dy}{e^y} \quad \text{or}$$

$$\Rightarrow e^x = -e^{-y} + C \Rightarrow e^x + e^{-y} = C$$

$$21. \quad xy' = y + x e^{y/x}$$

$$y' = \frac{y}{x} + e^{y/x} \quad y/x = v \Rightarrow x \frac{dv}{dx} + v = \frac{dv}{dx} \quad \text{or}$$

$$\Rightarrow x \frac{dv}{dx} + v = v + e^v \Rightarrow \frac{dv}{e^v} = \frac{dx}{x} \Rightarrow -e^{-v} = \ln|x| + C$$

$$\Rightarrow -e^{-y/x} = \ln|x| + C$$

$$22. \quad \frac{dy}{dx} = \frac{x^2-1}{y^2+1} \quad y(-1) = 1$$

$$(y^2+1)dy = (x^2-1)dx \Rightarrow \frac{y^3}{3} + y = \frac{x^3}{3} - x + C \quad \text{or}$$

$$\frac{1}{3} + 1 = -\frac{1}{3} + 1 + C \Rightarrow C = \frac{2}{3} \quad \therefore \frac{y^3}{3} + 3y = \frac{x^3}{3} - 3x + 8$$

$$23. \quad xy' + y - y^2 e^{2x} = 0$$

$$y' + \frac{1}{x}y = \frac{e^{2x}}{x}y^2 \Rightarrow y'y^{-2} + \frac{1}{x}y^{-1} = \frac{e^{2x}}{x} \quad \text{or}$$

$$\frac{1}{y} = v \Rightarrow -\frac{1}{y^2}y' = v' \Rightarrow -v' + \frac{1}{x}v = \frac{e^{2x}}{x}$$

$$v' - \frac{1}{x}v = \frac{-e^{2x}}{x} \quad \mu(x) = \exp \int -\frac{1}{x} dx = x^{-1}$$

$$\frac{1}{x}v' - \frac{1}{x^2}v = \frac{-e^{2x}}{x^2}$$

$$\left(\frac{y}{x}\right)' = \int -\frac{e^{2x}}{x^2} dx + C \quad \frac{y}{x^2} = -\int \frac{e^{2x}}{x^2} dx + C$$

$$24. \quad 2 \sin y \cot x \, dx + \cot y \sin x \, dy = 0$$

$$\frac{dy}{dx} = \frac{-2 \sin y \cot x}{\cot y \sin x} \Rightarrow \frac{dy}{\frac{1}{\tan y}} = -2 \cot y x \, dx$$

$$\Rightarrow \frac{\cot y \, dy}{\sin y} = -2 \frac{\cot x \, dx}{\sin x} \Rightarrow \ln \sin y = -2 \ln \sin x + \ln c$$

$$\Rightarrow \ln \sin y \sin^2 x = \ln c \Rightarrow \sin^2 x \sin y = c$$

$$25. \quad \left(2 \frac{x}{y} - \frac{y}{x^2+y^2}\right) dx + \left(\frac{x}{x^2+y^2} - \frac{x^2}{y^2}\right) dy = 0$$

$$M_y = -\frac{2x}{y^2} - \left(\frac{x^2+y^2-2y^2}{x^2+y^2}\right), \quad N_x = \frac{(x^2+y^2)-2x^2}{(x^2+y^2)^2} - \frac{2x}{y^2}$$

eq. is exact.

$$\psi_x(x, y) = 2 \frac{x}{y} - \frac{y}{x^2+y^2}, \quad \psi_y(x, y) = \frac{x}{x^2+y^2} - \frac{x^2}{y^2}$$

$$\psi(x, y) = \frac{x^2}{y} - \frac{1}{y} \frac{x}{y} + h(y), \quad \psi_y(x, y) = -\frac{x^2}{y^2} - \frac{-x/y^2}{(x/y)^2+1} + h'(y)$$

$$= \frac{x}{x^2+y^2} - \frac{x^2}{y^2} \Rightarrow h'(y) = 0 \Rightarrow h(y) = 0$$

$$\therefore \frac{x^2}{y} - \frac{1}{y} \frac{x}{y} = C$$

$$26. \quad (2y+1) dx + \left(\frac{x^2-y}{x}\right) dy = 0$$

$$(2xy+x)dx + (x^2-y)dy = 0 \quad M(x,y) = 2xy+x, \quad N(x,y) = x^2-y \quad : \text{J}$$

$$M_y = 2x, \quad N_x = 2x \quad \psi_x(x,y) = 2xy+x, \quad \psi_y(x,y) = x^2-y$$

$$\psi(x,y) = x^2y + \frac{1}{2}x^2 + h(y) \Rightarrow \psi_y(x,y) = x^2 + h'(y) = x^2 - y \Rightarrow h'(y) = -y$$

$$\Rightarrow h(y) = -\frac{1}{2}y^2 \quad \therefore x^2y + \frac{1}{2}x^2 - \frac{1}{2}y^2 = C$$

$$27. (\cos 2y - \sin x)dx - 2xy \sin 2y dy = 0$$

$$M_y = -2 \sin 2y, \quad N_x = -2(1+\frac{1}{2}x^2) \quad M_y \neq N_x \quad : \text{J}$$

$$\therefore \omega: (\cos x \cos 2y - \sin x \cos x)dx - 2 \sin x \sin 2y dy = 0$$

$$M_y = -2 \cos x \sin 2y, \quad N_x = -2 \cos x \sin 2y \quad M_y = N_x \Rightarrow \text{eq. is exact}$$

$$28. \frac{dy}{dx} = \frac{3x^2 - 2y - y^3}{2x + 3xy^2}$$

$$M_y = -2 - 3y^2, \quad N_x = -2 - 3y^2 \quad M_y = N_x \quad \text{eq. is exact} \quad : \text{J}$$

$$\psi_x(x,y) = 3x^2 - 2y - y^3, \quad \psi_y = -(2x + 3xy^2) \quad (*)$$

$$\psi(x,y) = x^3 - 2xy - xy^3 + h(y) \Rightarrow \psi_y(x,y) = -2x - 3xy^2 + h'(y) \\ \stackrel{(*)}{=} -(2x + 3xy^2)$$

$$\Rightarrow h'(y) = 0 \Rightarrow h(y) = 0 \quad \therefore x^3 - 2xy - xy^3 = C$$

$$29. \frac{dy}{dx} = \frac{2y + \sqrt{x^2 - y^2}}{2x}$$

$$y = xv \Rightarrow \frac{dy}{dx} = x \frac{dv}{dx} + v \Rightarrow x \frac{dv}{dx} + v = \frac{2xv + \sqrt{x^2 - x^2v^2}}{2x}$$

$$\frac{4dv}{2\sqrt{1-v^2}} = \frac{dx}{x} \Rightarrow \frac{4}{2} \sin^{-1} v = \ln|x| + C = \frac{(2x + \sqrt{1-v^2})}{2}$$

$$30. \frac{dy}{dx} = \frac{y^3}{1-2xy^2}, \quad y(0) = 1$$

$$y^3 dx + (2xy^2 - 1) dy = 0 \quad : \text{J}$$

$$M_y = 3y^2, N_x = 2y^2 \quad \frac{N_x - M_y}{M} = \frac{2y^2 - 3y^2}{y^3} = -\frac{1}{y} = Q(y)$$

$$\mu(y) = \exp \int -\frac{1}{t} dt = \frac{-1}{y} \quad y^2 dx + (2xy - y^{-1}) dy = 0$$

$$M_y = 2y, N_x = 2y \quad M_y = N_x \quad \psi_x(x, y) = y^2, \psi_y(x, y) = 2xy - y^{-1}$$

$$\psi(x, y) = xy^2 + h(y) \quad \psi_y(x, y) = 2xy + h'(y) = 2xy - y^{-1}$$

$$h'(y) = -y^{-1} \Rightarrow h(y) = -\ln|y| \quad \therefore xy^2 - \ln|y| = C$$

$$31. (x^2y + xy - y) dx + (x^2y - 2x^2) dy = 0$$

$$y - 2\ln|y| = x + \ln|x| + \frac{1}{x} + C \quad \frac{dy}{dx} = \frac{x^2y + xy - y}{x^2y - 2x^2} = \frac{(x^2 + x - 1)y}{x^2(y - 2)} \Rightarrow \frac{(y - 2)dy}{y} = \frac{(x^2 + x - 1)dx}{x^2}$$

$$y - 2\ln|y| = x + \ln|x| + \frac{1}{x} + C$$

$$32. \frac{dy}{dx} = -\frac{3x^2y + y^2}{2x^3 + 3xy} \quad y(1) = -2$$

$$(2x^3 + 3xy) dy + (3x^2y + y^2) dx = 0$$

$$M(x, y) = 3x^2y + y^2, N(x, y) = 2x^3 + 3xy$$

$$M_y = 3x^2 + 2y, N_x = 6x^2 + 3y \quad \frac{N_x - M_y}{M} = \frac{6x^2 + 3y - 3x^2 - 2y}{3x^2y + y^2}$$

$$= \frac{3x^2 + y}{3x^2y + y^2} = \frac{1}{y} \quad \mu(y) = \exp \int \frac{1}{t} dt = e^{\ln y} = y$$

$$\Rightarrow (2x^3y + 3xy^2) dy + (3x^2y^2 + y^3) dx = 0 \Rightarrow d(x^3y^2 + xy^3) = 0$$

$$\Rightarrow x^3y^2 + xy^3 = C \quad \& \quad 4 + (-2)^3 = C \Rightarrow C = -4$$

$$x^3y^2 + xy^3 = -4$$

$$33. \text{داده شده هر معادله به صورت } y = G(x) \text{ را در } \phi = \frac{dy}{dx} \text{ قرار دهیم}$$

می توان به طریق زیر عمل کرد.

(الف) نسبت به x مشتق می گیریم، (ب) با جمل معادله قسمت (الف) را به صورت ϕ جایگزین می کنیم

$$\text{می آوریم. (ج) معادله} \quad x(y')^2 - xy' - y = 0 \text{ را حل کنید.}$$

این رابطه و معادله اصلی $y = G(p)$ ، جواب را به صورت $y = G(p)$ می‌دهد.

$$y = G(p), \quad p = \frac{dy}{dx}$$

$$p = \frac{dy}{dx} = \frac{dG(p)}{dx} = \frac{dG(p)}{dp} \frac{dp}{dx} = G'(p) \cdot \frac{dp}{dx} \Rightarrow$$

$$dx = \frac{G'(p)}{p} dp \Rightarrow x = \int \frac{G'(p)}{p} dp + c$$

3.4. معادله دفرانسیل $y - \ln p = 0$ را که در آن $p = \frac{dy}{dx}$ است، با روش مشابه 3.3 حل کنید. این معادله را مستقیماً نیز حل کنید و نتایج را مقایسه کنید که هر دو جواب یکی است.

$$\ln y' = y \quad \frac{1}{y} = \ln p, \quad p = \frac{dy}{dx}$$

$$y' = p \cdot \frac{1}{p} \Rightarrow p = p' \cdot \frac{1}{p} \Rightarrow p^2 = \frac{dp}{dx} \Rightarrow dx = \frac{dp}{p^2} \Rightarrow x = -\frac{1}{p}$$

$$\frac{dy}{dx} = -\frac{1}{x} \Rightarrow dy = -\frac{dx}{x} \Rightarrow y = -\ln x + c \quad y = \ln p$$

$$e^y = p = \frac{dy}{dx} \Rightarrow e^y dy = dx \Rightarrow x = -e^y + c$$

3.5. معادله $\frac{dy}{dx} = g_1(x) + g_2(x)y + g_3(x)y^2$ مشهور به معادله ریکتی است. فرض کنید که یک جواب مفروضی $y_1(x)$ این معادله معلوم باشد. جواب عمومی دیگری را که می‌توان با تبدیل زیر بدست آورد.

$$y = y_1(x) + \frac{1}{v(x)}$$

نشان دهید که $v(x)$ در معادله خطی مرتبه اول

$$\frac{dv}{dx} = -(g_2 + 2g_3 y_1) v - g_3$$

صدق می‌کند. توجه شود که $y_1(x)$ تنها یک جواب دلخواه است.

$$\frac{dy}{dx} = \frac{dy_1}{dx} + \frac{v'(x)}{v(x)^2}$$

$$\frac{dy_1}{dx} + \frac{v'(x)}{v(x)^2} = g_1(x) + g_2(x) \left(y_1 + \frac{1}{v} \right) + g_3(x) \left(y_1 + \frac{1}{v} \right)^2$$

$$\frac{dy_1}{dx} - \frac{v'(x)}{v(x)^2} = \cancel{q_1(x)} + \cancel{q_2(x)y_1} + \frac{q_2(x)}{v(x)} + \cancel{q_3(x)y_1^2} + 2 \frac{q_3(x)y_1}{v(x)} + \frac{q_3(x)}{v(x)^2}$$

$$\therefore \frac{dv}{dx} = -(q_2 + 2q_3 y_1)v - q_3$$

36. با استفاده از روش متغیر 35 و جواب خصوصی داده شده، حرکت از سمت بالا را برای زیر را حل کنید.

$$y' = 1 + x^2 - 2xy + y^2 \quad ; \quad y_1(x)$$

$$y = x + \frac{1}{v(x)} \quad y' = 1 - \frac{v'(x)}{v(x)^2} \Rightarrow 1 - \frac{v'(x)}{v(x)^2} = 1 + x^2 - 2x(x + \frac{1}{v(x)})$$

$$+ (x + \frac{1}{v(x)})^2 \Rightarrow -\frac{v'(x)}{v(x)^2} = \cancel{x^2} - \frac{2x}{\cancel{v(x)}} + \cancel{x^2} + \frac{2x}{\cancel{v(x)}} + \frac{1}{v(x)^2}$$

$$\Rightarrow -v'(x) = 1 \Rightarrow v(x) = -x + C$$

$$\therefore y = x + \frac{1}{C-x} \quad y = x + (C-x)^{-1}$$