

$y' = 2 \tan x \sec x - (\sin x) y^2$ فرض: $y_1 = \sec x$ را در صورتی که y_1 یک جواب است بیابیم
 حل:

$$y = y_1 + \frac{1}{z} = \sec x + \frac{1}{z}$$

$$y' = \sec x \tan x - \frac{1}{z^2} z'$$

$$\sec x \tan x - \frac{z'}{z^2} = 2 \sec x \tan x - \sin x \left(\sec x + \frac{1}{z} \right)^2$$

$$-\frac{z'}{z^2} = \sec x \tan x - \sin x \left(\sec^2 x + \frac{2 \sec x}{z} + \frac{1}{z^2} \right)$$

$$-\frac{z'}{z^2} = -\frac{2}{z} \tan x - \frac{\sin x}{z^2}$$

$$z' = 2z \tan x - \sin x \quad \mid \quad z' = -2(\tan x)z = -\sin x$$

$$\mu = e^{\int -2 \tan x dx} = e^{-2 \ln \cos x} = e^{\ln \cos^2 x} = \cos^2 x$$

$$(\cos^2 x) z' - 2(\tan x \cos^2 x) z = -\sin x \cos^2 x$$

$$[(\cos^2 x) z]' = -\sin x \cos^2 x$$

$$\Rightarrow (\cos^2 x) z = -\int \sin x \cos^2 x dx = -\frac{\cos^3 x}{3} + C$$

$$\Rightarrow z = \frac{\cos x}{3} + \frac{C}{\cos^2 x} = \frac{\cos x}{3} + C \sec^2 x$$

پس: $y = y_1 + \frac{1}{z} = \sec x + \frac{\cos x}{3} + C \sec^2 x$

(claimant eq.) معادله کلید

معادله به شکل $y - xy' = f(y')$ است معادله کلید به این شکل

$$y'' - y' - xy'' = \frac{d}{dx} f(y') = \frac{d}{dy} f(y') \cdot \frac{dy'}{dx} = f'(y') \cdot y''$$

$$\Rightarrow y'' [f'(y') + x] = 0 \quad y'' = 0 \Rightarrow y = ax + b$$

$$y - xy' = f(y') \Rightarrow ax + b - x(a) = f(a) \Rightarrow b = f(a)$$

$$y = ax + b = ax + f(a) \quad \text{جواب عمومی}$$

$$f'(y') + x = 0 \Rightarrow \begin{cases} x = -f'(y') \\ y = xy' + f(y') \end{cases} \quad \text{جواب منفرد (جوابی بیرونی)}$$

Ex. $y = xy' + (y')^2$

$$y' = xy'' + y' + 2y' \cdot y'' \Rightarrow y''(x + 2y') = 0$$

$$y'' = 0 \Rightarrow y = ax + b \quad \text{نکته: } ax + b = x \cdot a + (a)^2 \Rightarrow b = a^2$$

$$\Rightarrow y = ax + a^2 \quad \text{جواب عمومی}$$

$$x + 2y' = 0 \Rightarrow y' = -\frac{x}{2} \quad \text{نکته: } y = x \left(-\frac{x}{2}\right) + \left(-\frac{x}{2}\right)^2 \\ = -\frac{x^2}{2} + \frac{x^2}{4} = -\frac{x^2}{4}$$

$$y = -\frac{x^2}{4} \quad \text{جواب منفرد}$$

1) (b) $xy'' + y' = 1 \quad x > 0$ page 86 $y'' = f(x, y)$

$v = y' \Rightarrow v' = y'' \quad x v' + v = 1 \Rightarrow v' + \frac{1}{x} v = \frac{1}{x}$

$\mu(x) = \int \frac{1}{x} dx = e^{\ln x} = x \rightarrow (xv)' = 1 \Rightarrow xv = x + C_1$

$v = 1 + C_1 x^{-1} \Rightarrow y' = 1 + C_1 x^{-1} \Rightarrow \frac{dy}{dx} = 1 + C_1 x^{-1} \Rightarrow dy = (1 + C_1 x^{-1}) dx$

$y = x + C_1 \ln x + C_2$

(c) $y'' + x(y')^2 = 0 \Rightarrow v' + x v^2 = 0$

$\frac{dv}{dx} + x v^2 = 0 \Rightarrow \frac{dv}{v^2} = -x dx \Rightarrow -\frac{1}{v} = -\frac{1}{2} x^2 + \frac{C_1}{2}$

$\frac{1}{v} = \frac{1}{2} x^2 - \frac{C_1}{2} \Rightarrow \frac{2}{v} = x^2 - C_1 \Rightarrow \frac{2}{y'} = x^2 - C_1$

$\frac{y'}{2} = \frac{1}{x^2 - C_1} \Rightarrow \frac{dy}{dx} = \frac{2}{x^2 - C_1} = \frac{1}{C_1} \left(\frac{1}{x - C_1} - \frac{1}{x + C_1} \right)$

$dy = \frac{1}{C_1} \left(\frac{1}{x - C_1} - \frac{1}{x + C_1} \right) dx \Rightarrow y = \frac{1}{C_1} \ln \frac{x - C_1}{x + C_1} + C_2$

2) $y'' = f(y, y') \quad v = y' \Rightarrow v' = y'' \Rightarrow v' = f(y, v)$

$\frac{dv}{dx} = v' = \frac{dv}{dy} \frac{dy}{dx} = v \frac{dv}{dy} \Rightarrow v \frac{dv}{dy} = f(y, v)$

(b) $y'' + y = 0 \quad y' = v \Rightarrow y'' = v' = v \frac{dv}{dy}$

$v \frac{dv}{dy} + y = 0 \Rightarrow v dv + y dy = 0 \Rightarrow v^2 + y^2 = C_1$

$y'^2 + y^2 = C_1 \Rightarrow \left(\frac{dy}{dx} \right)^2 + y^2 = C_1 \Rightarrow \frac{dy}{dx} = \pm \sqrt{C_1 - y^2}$

$\frac{dy}{dx} = \sqrt{C_1 - y^2} \Rightarrow \frac{dy}{\sqrt{C_1 - y^2}} = dx$

$\frac{dy}{dx} = \sqrt{C_1 - y^2} \Rightarrow \frac{1}{\sqrt{C_1 - y^2}} = \frac{1}{C_1} \sin^{-1} \frac{y}{C_1} = x + C_2$

$$(a) \quad y y'' + y'^2 = 0 \quad y' = v \quad y'' = v' = \frac{dv}{dx} = \frac{dv}{dy} \frac{dy}{dx} = v \frac{dv}{dy}$$

$$y v \frac{dv}{dy} + v^2 = 0 \Rightarrow y \frac{dv}{dy} + v = 0 \Rightarrow \frac{dv}{v} + \frac{dy}{y} = 0$$

$$\ln v + \ln y = \ln \frac{C_1}{2} \Rightarrow v y = \frac{C_1}{2} \quad v = \frac{C_1}{2y}$$

$$\frac{dy}{dx} = \frac{C_1}{2y} \Rightarrow 2y dy = C_1 dx \Rightarrow y^2 = C_1 x + C_2$$

$$(3) (a) \quad y' y'' = 2 \quad y(0) = 1, \quad y'(0) = 2$$

$$y' = v \Rightarrow v v' = 2 \quad y'' = f(x, y') = f(y, y')$$

$$v \frac{dv}{dx} = 2 \Rightarrow v dv = 2 dx \Rightarrow \frac{1}{2} v^2 = 2x + C_1$$

$$v^2 = 4x + 2C_1 \Rightarrow \left(\frac{dy}{dx} \right)^2 = 4x + 2C_1 \quad 2 = 4(0) + 2C_1$$

$$4 = 4(0) + 2C_1 \quad C_1 = 0 \quad \left(\frac{dy}{dx} \right)^2 = 4x$$

$$\frac{dy}{dx} = \pm 2\sqrt{x} \quad dy = \pm 2\sqrt{x} dx \quad y = \pm \frac{4}{3} x^{3/2} + C_2$$

$$y(0) = 1 \Rightarrow 1 = 0 + C_2 \Rightarrow C_2 = 1$$

$$y = \pm \frac{4}{3} x^{3/2} + 1$$